UNIVERSITÀ DEGLI STUDI DELLA BASILICATA

Corso di Visione e Percezione

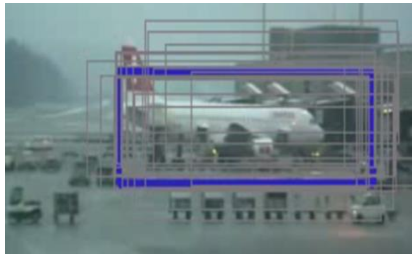
Omografie

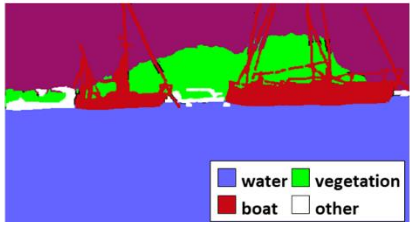
Docente

Domenico D. Bloisi

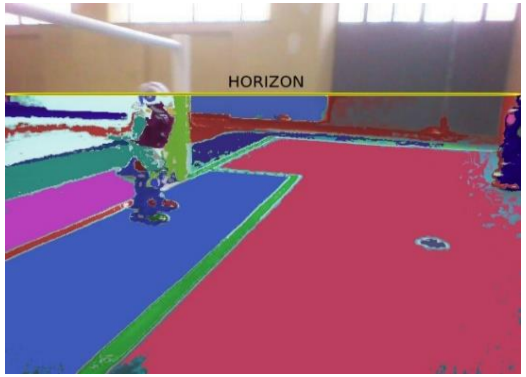








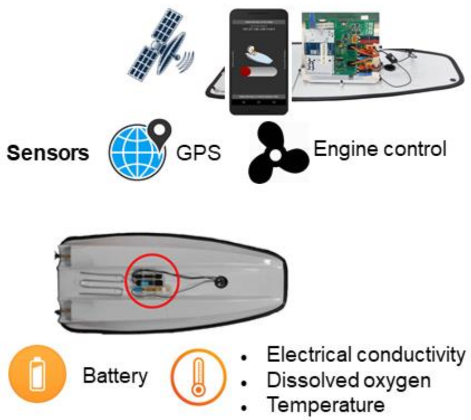


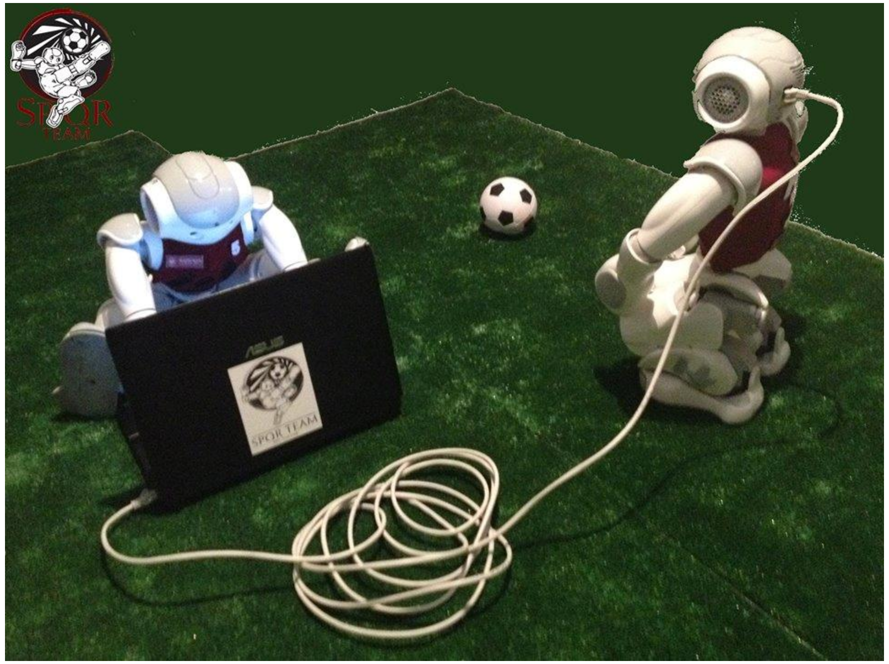



Domenico Daniele Bloisi

- Professore Associato
  Dipartimento di Matematica, Informatica ed Economia
  Università degli studi della Basilicata
  http://web.unibas.it/bloisi

- SPQR Robot Soccer Team
  Dipartimento di Informatica, Automatica e Gestionale Università degli studi di Roma "La Sapienza"
  http://spqr.diag.uniroma1.it





UNIBAS Wolves https://sites.google.com/unibas.it/wolves

- UNIBAS WOLVES is the robot soccer team of the University of Basilicata. Established in 2019, it is focussed on developing software for NAO soccer robots participating in RoboCup competitions.

- UNIBAS WOLVES team is twinned with SPQR Team at Sapienza University of Rome.

Informazioni sul corso

- Home page del corso: https://web.unibas.it/bloisi/corsi/visione-e-percezione.html
- Docente: Domenico Daniele Bloisi
- Periodo: II semestre marzo 2022 – giugno 2022

  - Martedì dalle 15:00 alle 17:00 (Aula Copernico)
  - Mercoledì dalle 8:30 alle 10:30 (Aula Copernico)

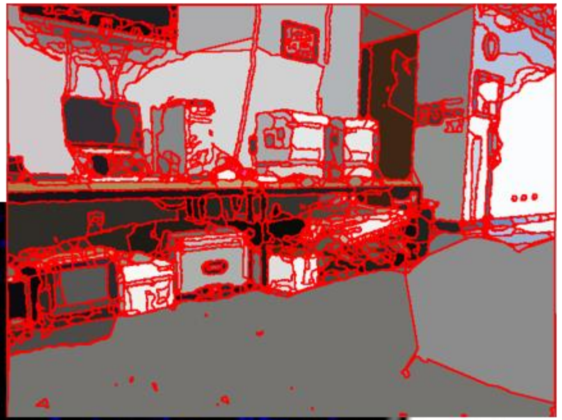
Ricevimento

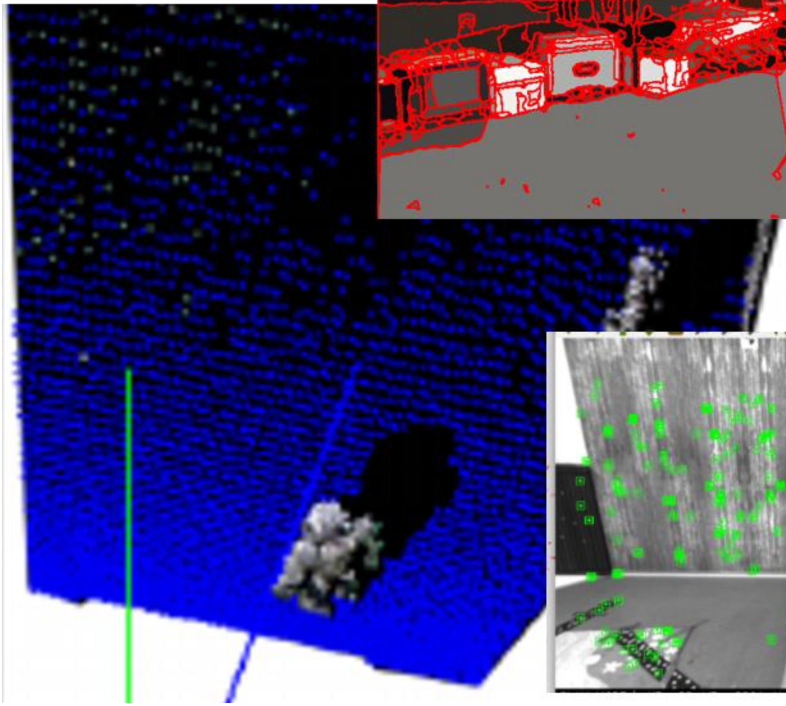
- Durante il periodo delle lezioni:
  Mercoledì dalle 11:00 alle 12:30 → Edificio 3D, II piano, stanza 15
  Si invitano gli studenti a controllare regolarmente la bacheca degli avvisi per eventuali variazioni
- Al di fuori del periodo delle lezioni:
  da concordare con il docente tramite email

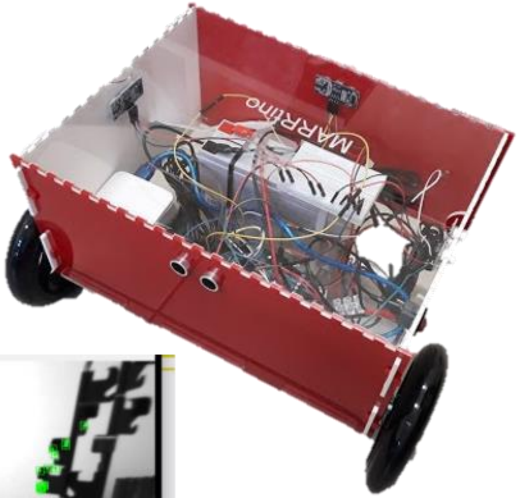
Per prenotare un appuntamento inviare una email a
domenico.bloisi@unibas.it

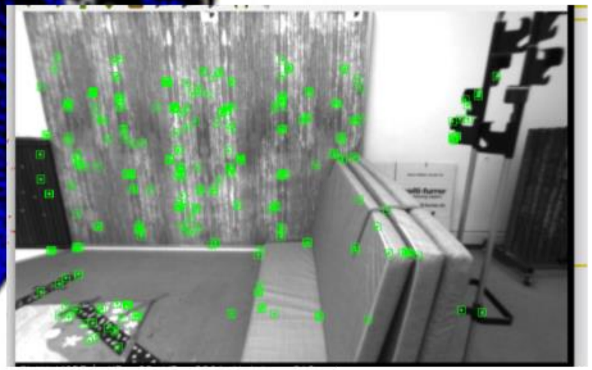
Programma – Visione e Percezione

- Introduzione al linguaggio Python
- Elaborazione delle immagini con Python
- Percezione 2D – OpenCV
- Introduzione al Deep Learning
- ROS
- Il paradigma publisher and subscriber
- Simulatori
- Percezione 3D - PCL









Riferimenti

- Queste slide sono adattate da
  - Noah Snavely - CS5670: Computer Vision
    "Lecture 7: Transformations and warping“
  - M. Brown and D. G. Lowe
    Recognising Panoramas
- I contenuti fanno riferimento al capitolo 3 del libro "Computer Vision: Algorithms and Applications" di Richard Szeliski, disponibile al seguente indirizzo http://szeliski.org/Book/

Image alignment

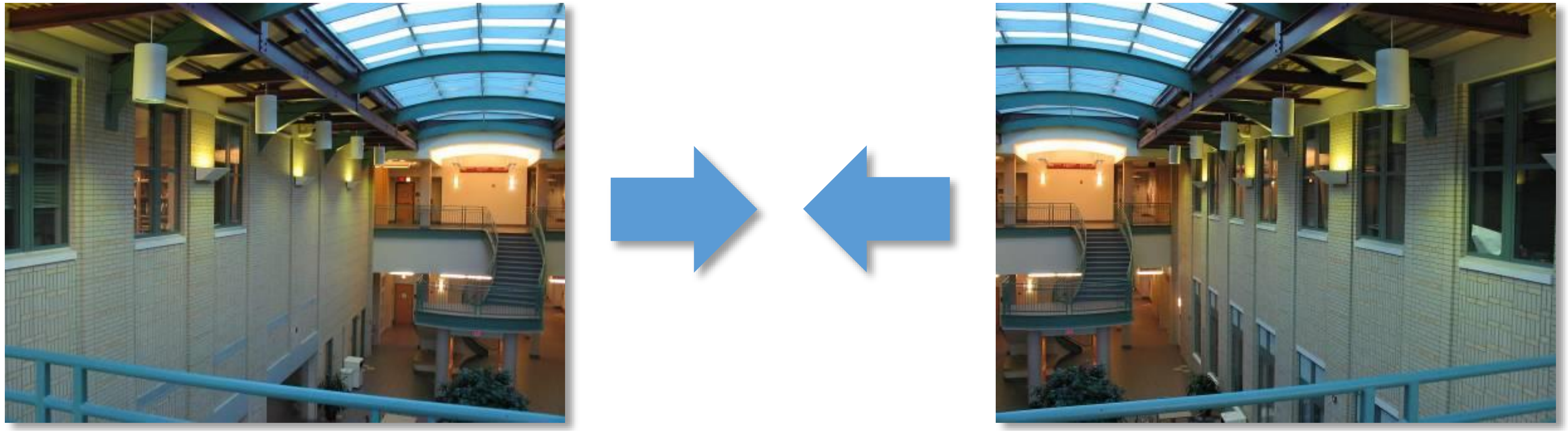


Image alignment

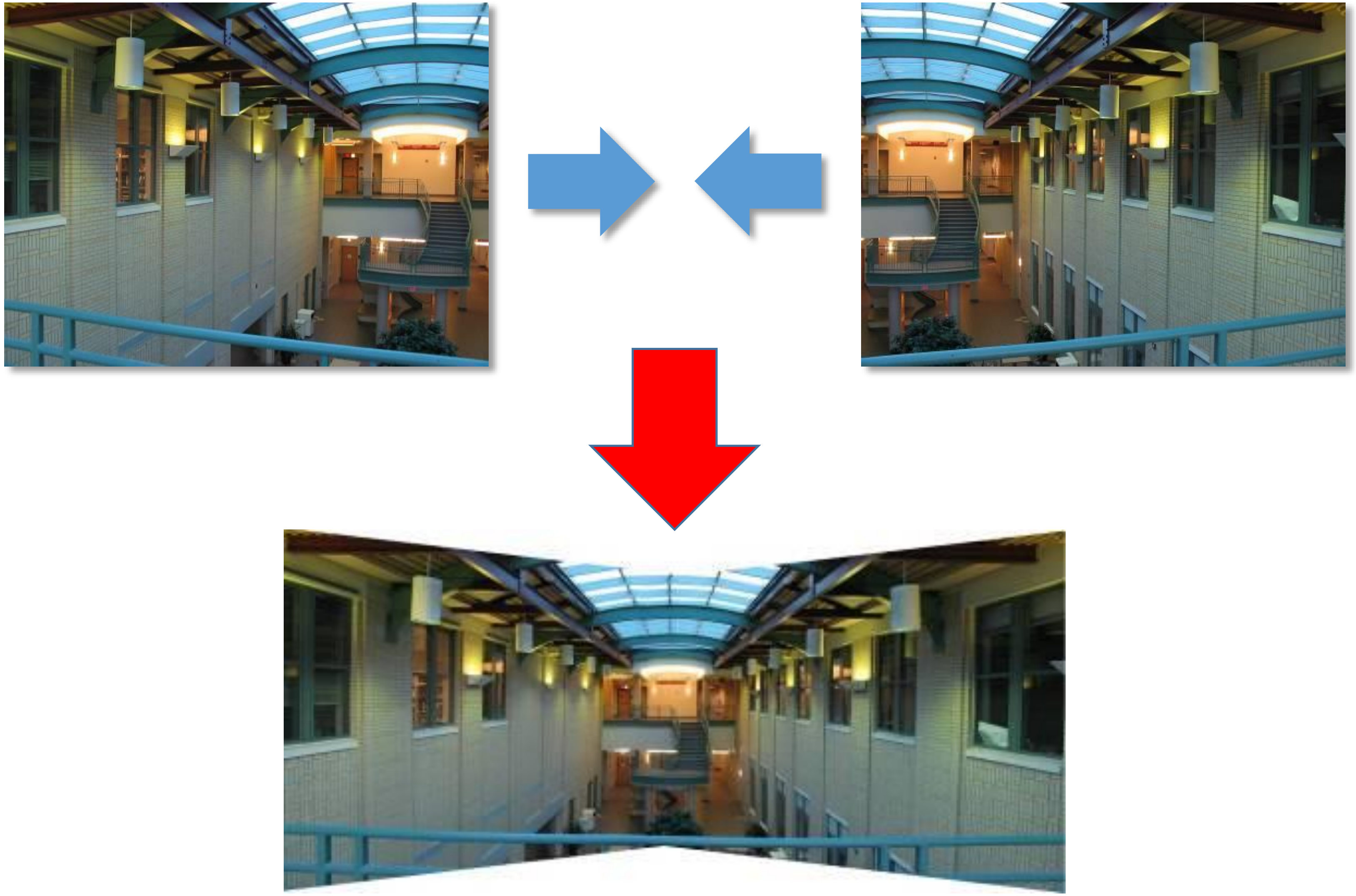


Image alignment

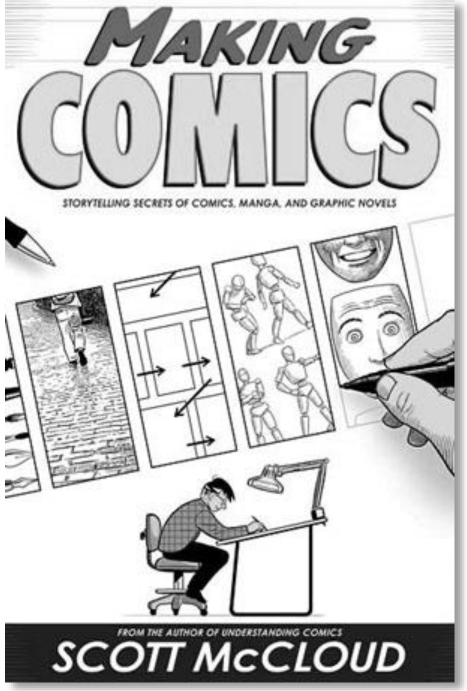
Come otteniamo questo risultato?

Sovrapposizione

Non è un buon risultato!

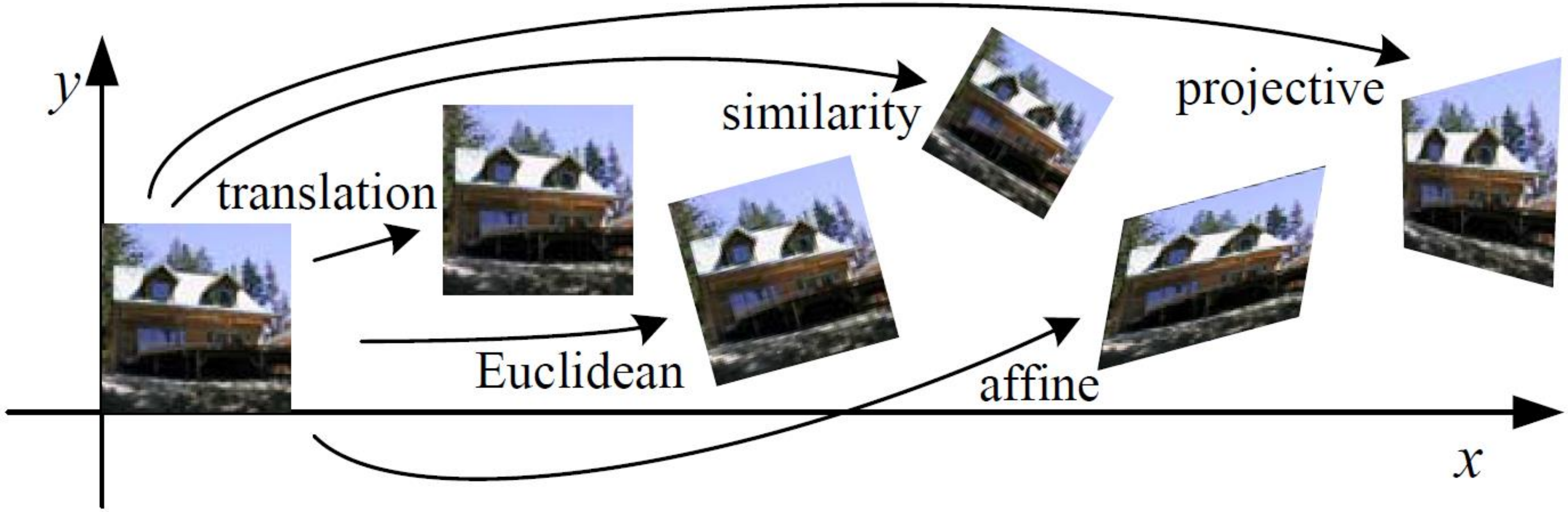
Image Transformation

What is the geometric relationship between these two images?



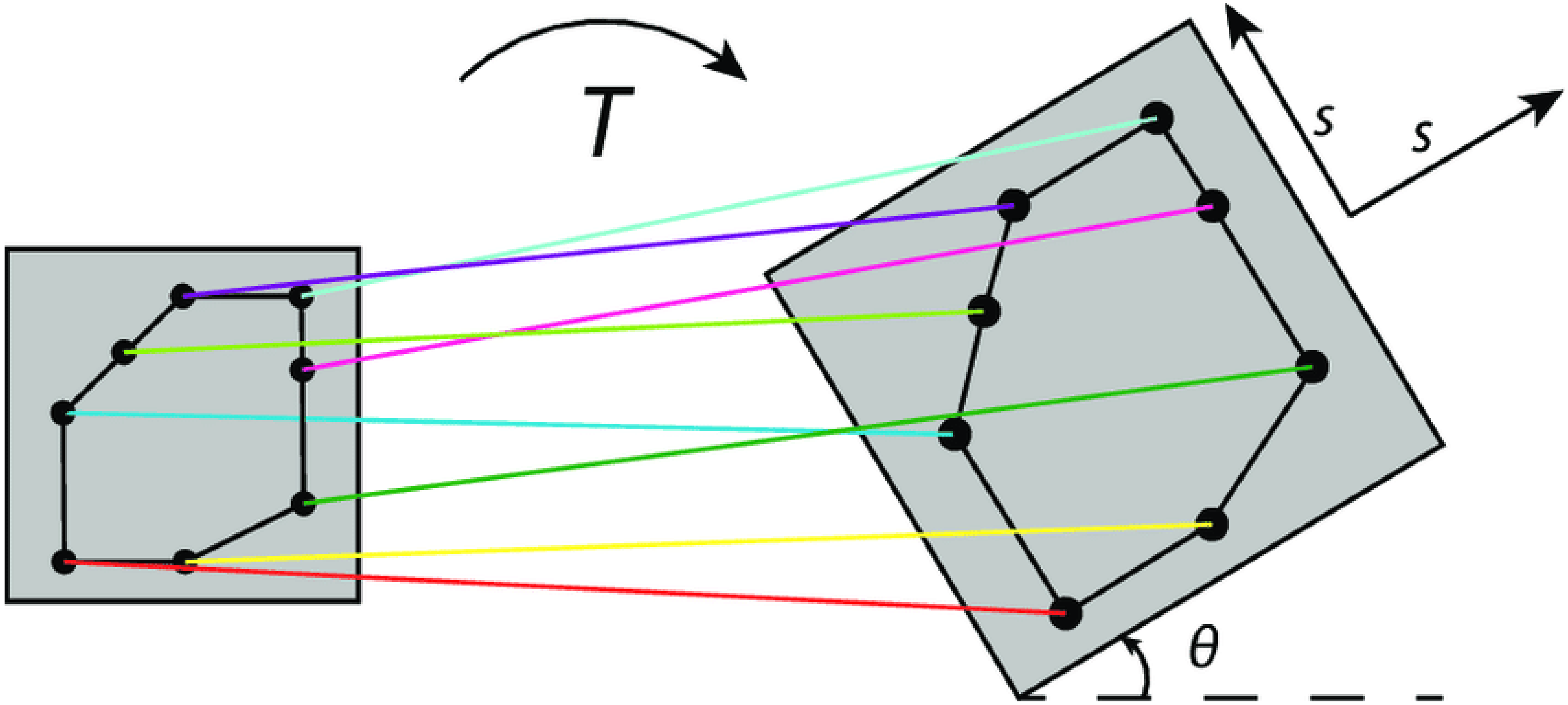
?

2D geometric image transformations



Similarity Transformation

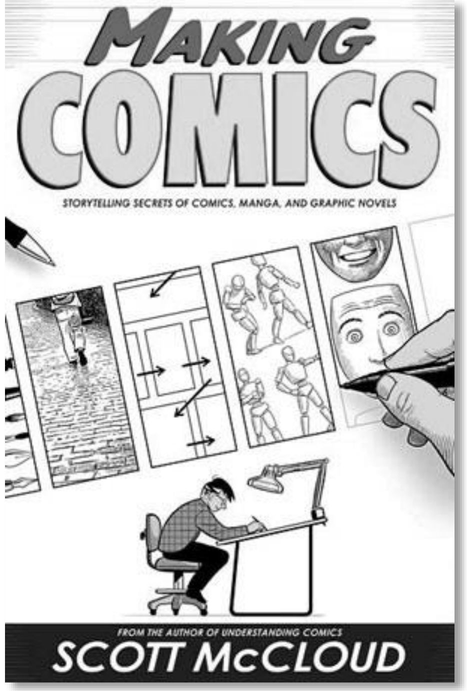
Translation + rotation + uniform scale



Source: Tobias Pahlberg

Similarity Transformation

What is the geometric relationship between these two images?

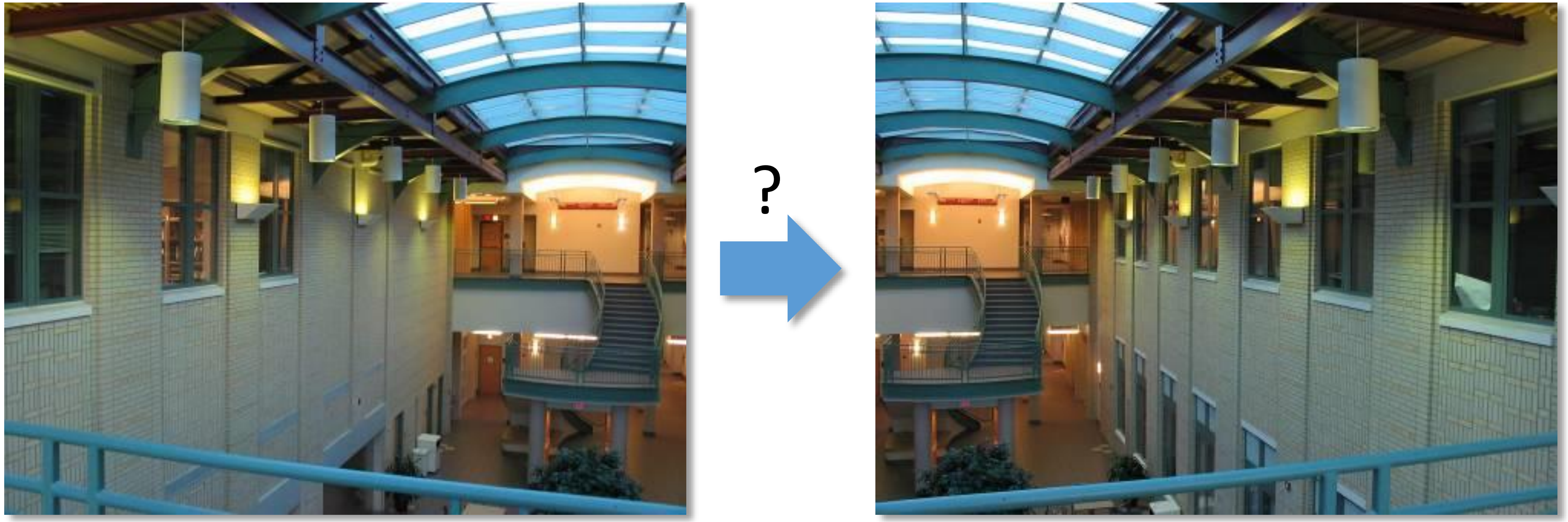


?

Answer: Similarity transformation (translation, rotation, uniform scale)

Similarity?

What is the geometric relationship between these two images?



Similarity?

Non è una trasformazione simile!

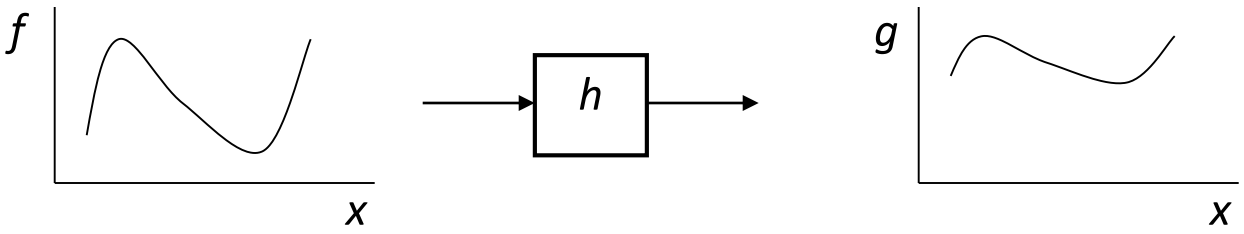
Image Mosaicing

1. First, we need to know what this transformation is.
2. Second, we need to figure out how to compute it using feature matches.

Image Filtering

- image filtering: change *range* of image

$$g(x) = h(f(x))$$



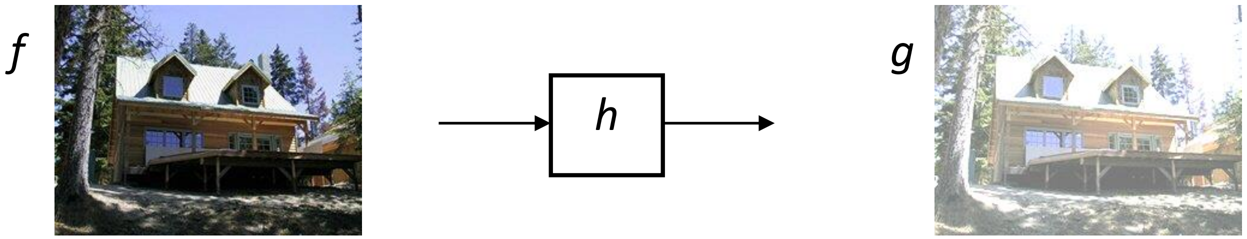
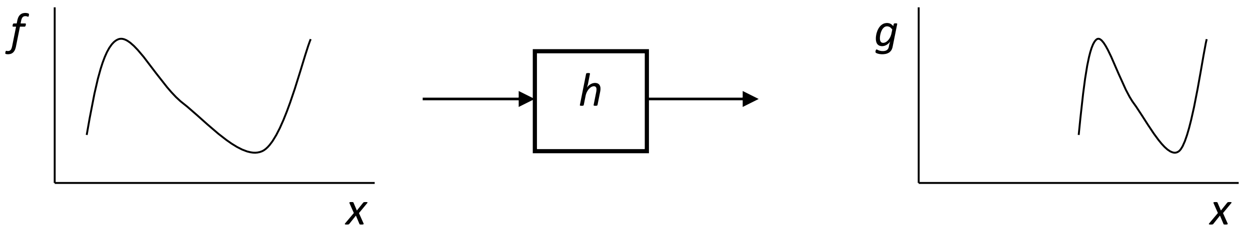
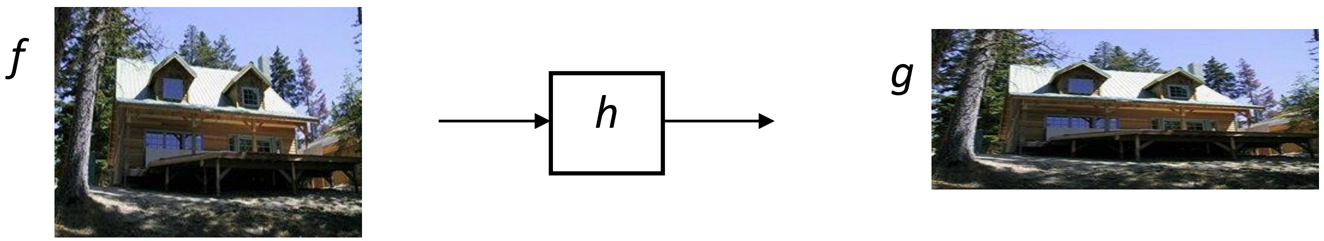


Image Warping

- image warping: change *domain* of image

$$g(x) = f(h(x))$$





Parametric (global) warping

- Examples of parametric warps:

translation

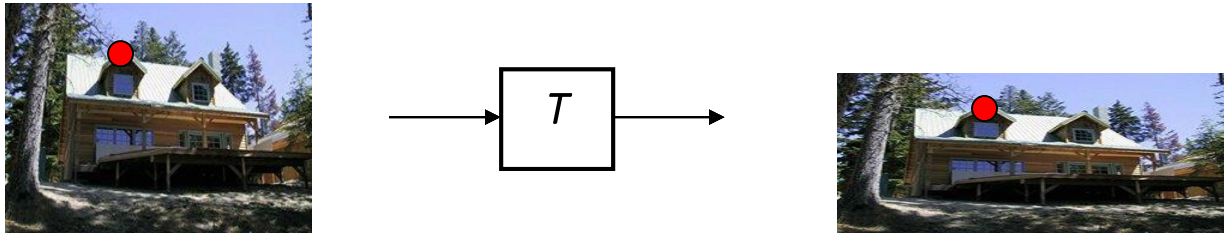
rotation

aspect

affine

perspective

Parametric (global) warping



p = (x,y) **p’** = (x’,y’)

Transformation *T* is a coordinate-changing machine:

$\mathbf{p}' = T(\mathbf{p})$

What does it mean that *T* is global?

- is the same for any point **p**
- can be described by just a few numbers (parameters)

Linear transforms

Let's consider *linear* transforms

(can be represented by a 2x2 matrix):

$$\mathbf{p}' = \mathbf{T}\mathbf{p}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{T} \begin{bmatrix} x \\ y \end{bmatrix}$$

Scaling

Uniform scaling by *s*:

(0,0)

(0,0)

$$\mathbf{S} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}$$

Rotation

Rotation by angle θ (about the origin)

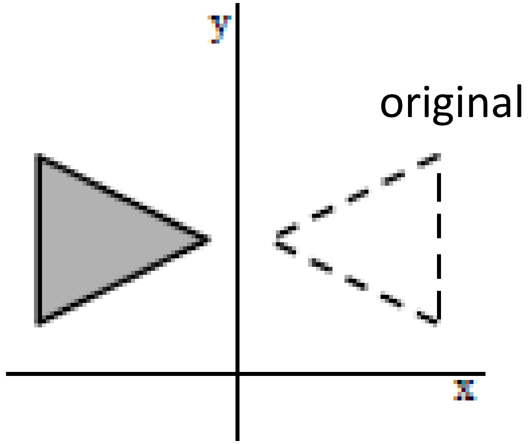
$$\mathbf{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

2x2 Matrices

What types of transformations can be represented with a 2x2 matrix?

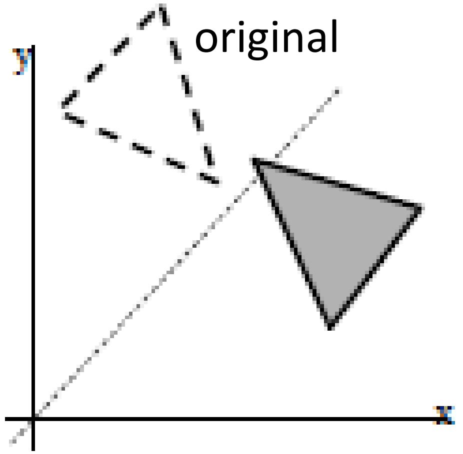
2D mirror about Y axis?

$$\begin{aligned} x' &= -x \\ y' &= y \end{aligned} \qquad \mathbf{T} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$



2D mirror across line y = x?

$$\begin{aligned} x' &= y \\ y' &= x \end{aligned} \qquad \mathbf{T} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



2x2 Matrices

What types of transformations can be represented with a 2x2 matrix?

2D Translation?

$$\begin{aligned} x' &= x + t_x \\ y' &= y + t_y \end{aligned}$$

NO!

Translation is not a linear operation on 2D coordinates

All 2D Linear Transformations

Linear transformations are combinations of …

- Scale,
- Rotation,
- Shear, and
- Mirror

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Properties of linear transformations:

- Origin maps to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Homogeneous Coordinates

How can we represent translation

$$\begin{aligned} x' &= x + t_x \\ y' &= y + t_y \end{aligned}$$

as a 3x3 matrix?

$$\begin{bmatrix} x \\ y \end{bmatrix} \xrightarrow{\text{homogeneous coords}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

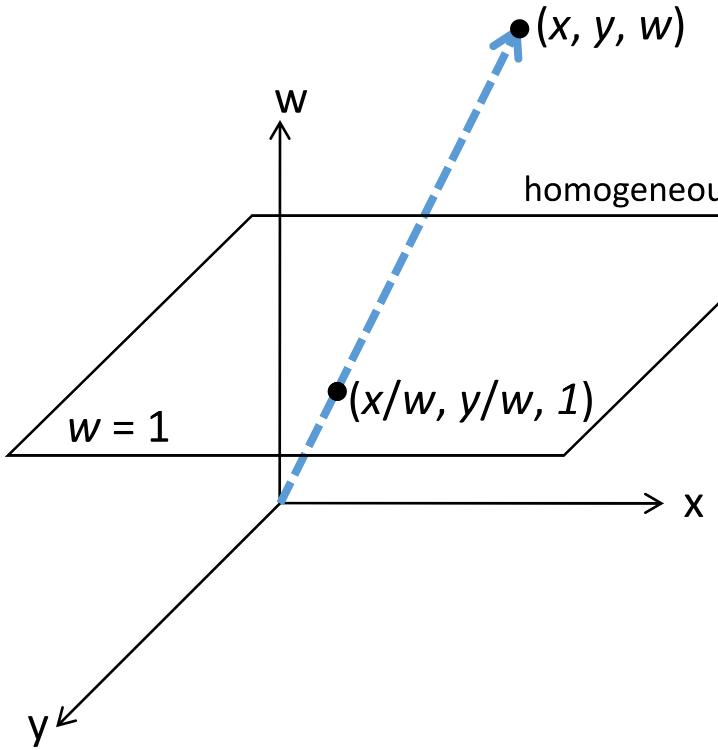
represent coordinates in 2 dimensions with a 3-vector

Homogeneous coordinates

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates



Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

Translation

Solution using homogeneous coordinates

$$\mathbf{T} = \left[\begin{array}{ccc} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{array}\right] \left[\begin{array}{c} x \\ y \\ 1 \end{array}\right] = \left[\begin{array}{c} x + t_x \\ y + t_y \\ 1 \end{array}\right]$$

Affine transformations

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ \boxed{0 \quad 0 \quad 1} \end{bmatrix}$$

any transformation represented by a 3x3 matrix with last row [0 0 1] we call an *affine transformation*

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}$$

Affine transformations

Affine transformations are combinations of …

- Linear transformations, and
- Translations

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Properties of affine transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

Basic affine transformations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} \boldsymbol{x}' \\ \boldsymbol{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{s}_x & 0 & 0 \\ 0 & \boldsymbol{s}_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2D *in-plane* rotation

$$\begin{bmatrix} \boldsymbol{x}' \\ \boldsymbol{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & \boldsymbol{sh}_x & 0 \\ \boldsymbol{sh}_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Shear

É una trasformazione affine?

NO

- Parallel lines do not necessarily remain parallel
- Ratios are not preserved

Trasformazioni non affini

$$\begin{bmatrix} a & b & c \\ d & e & f \\ \boxed{0 \quad 0 \quad 1} \end{bmatrix}$$

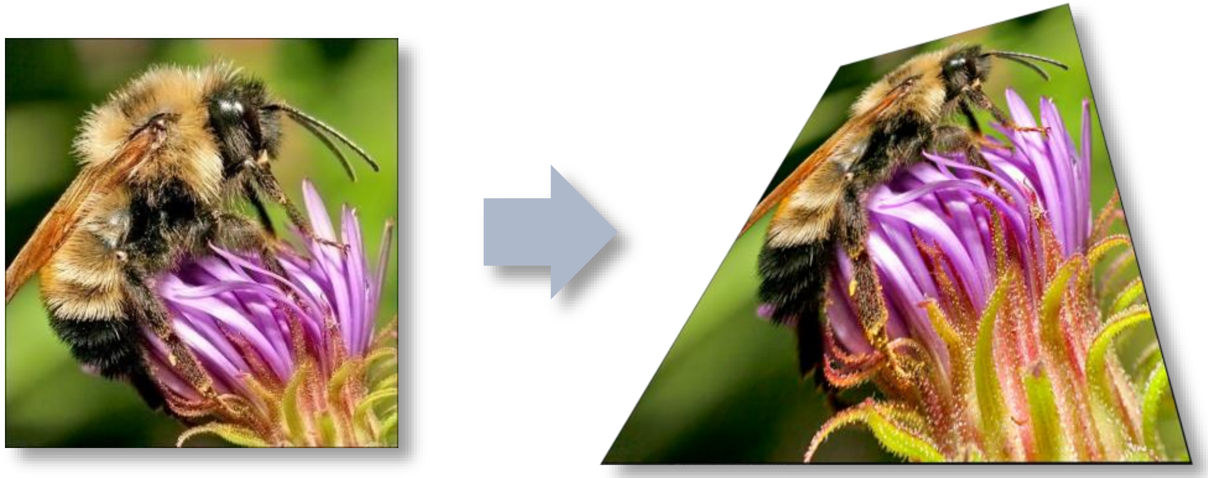
affine transformation

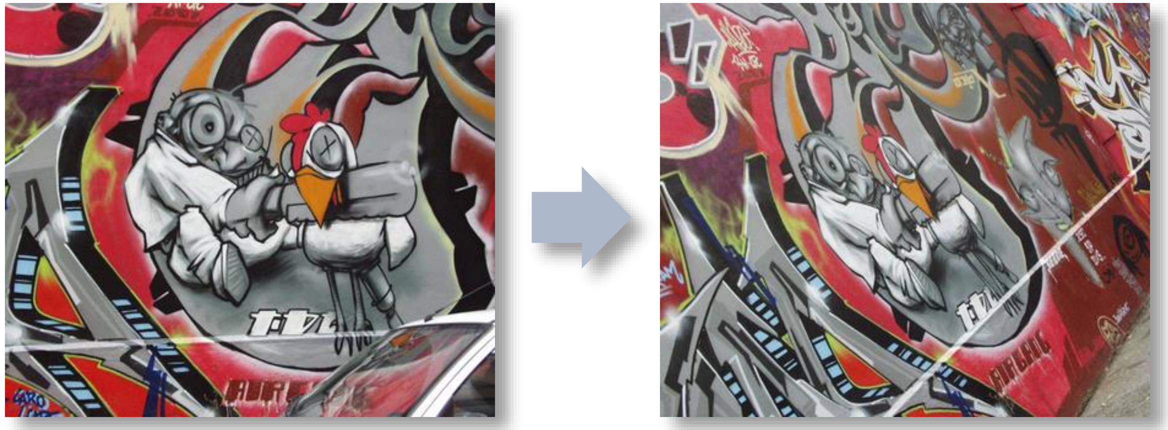
Cosa accade se cambiamo gli elementi della terza riga?

Omografie

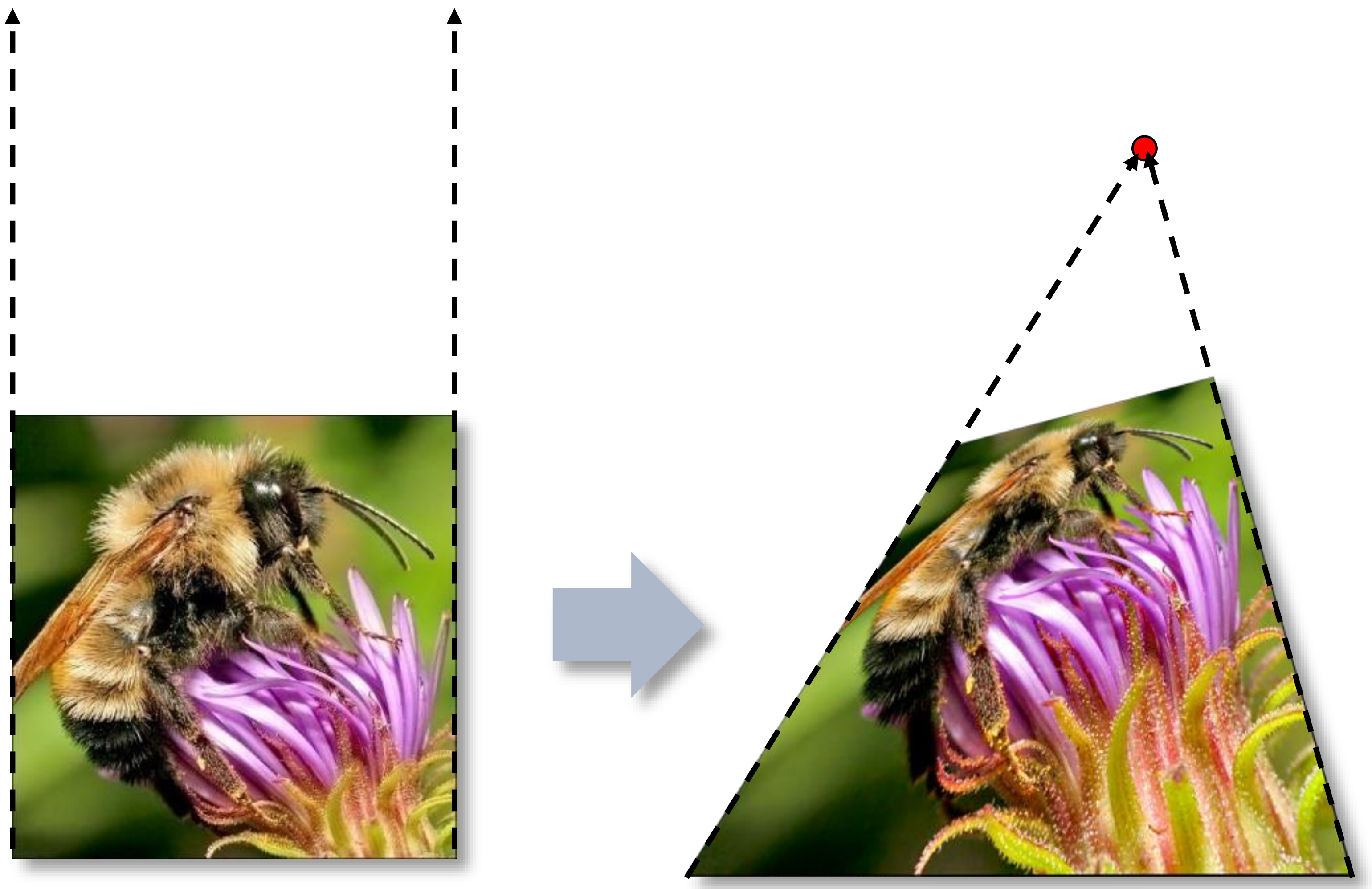
$$\mathbf{H} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix}$$

H is a *homography* (or *planar perspective map*)





Punti all’infinito



Top view con omografie

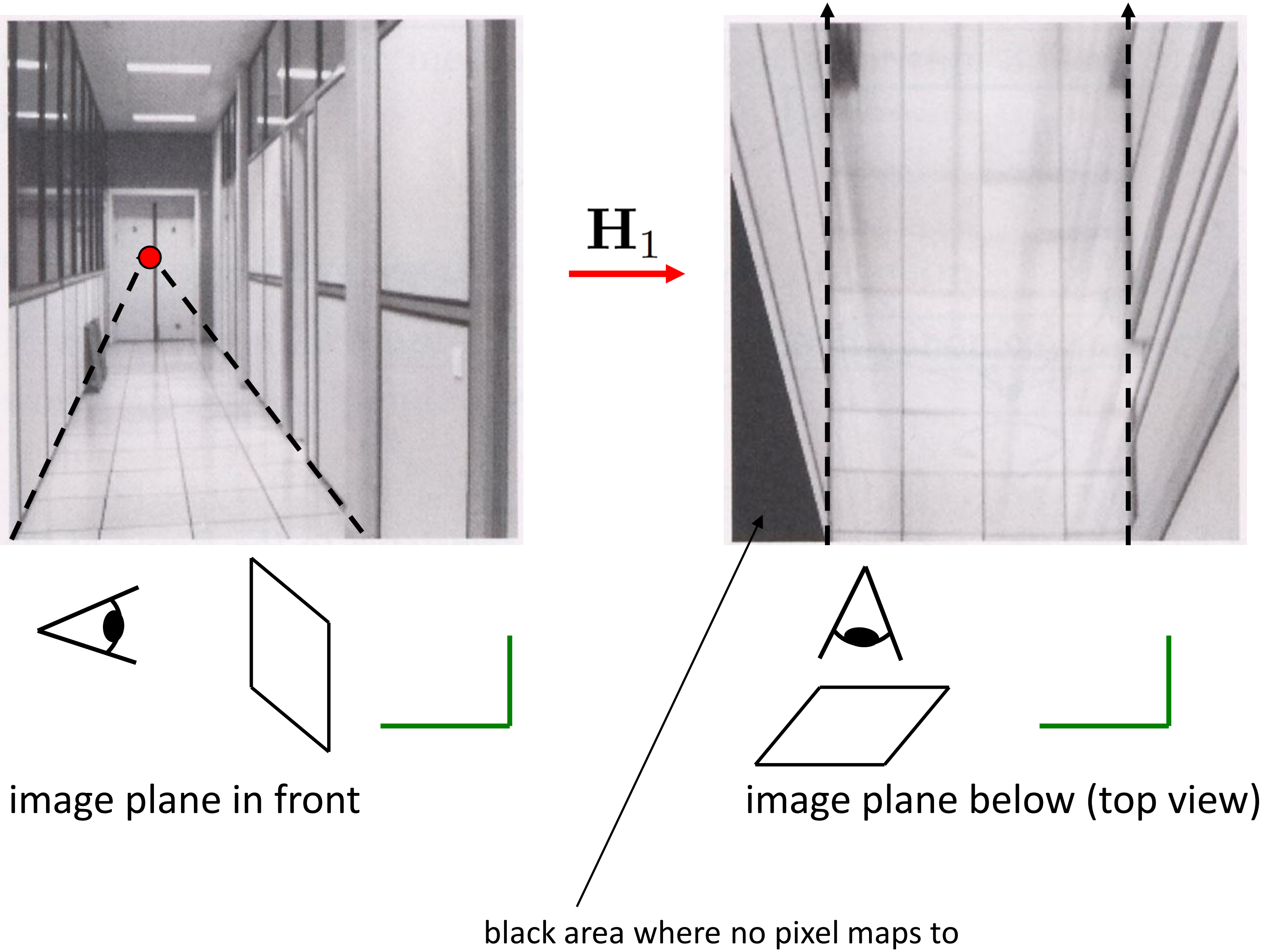
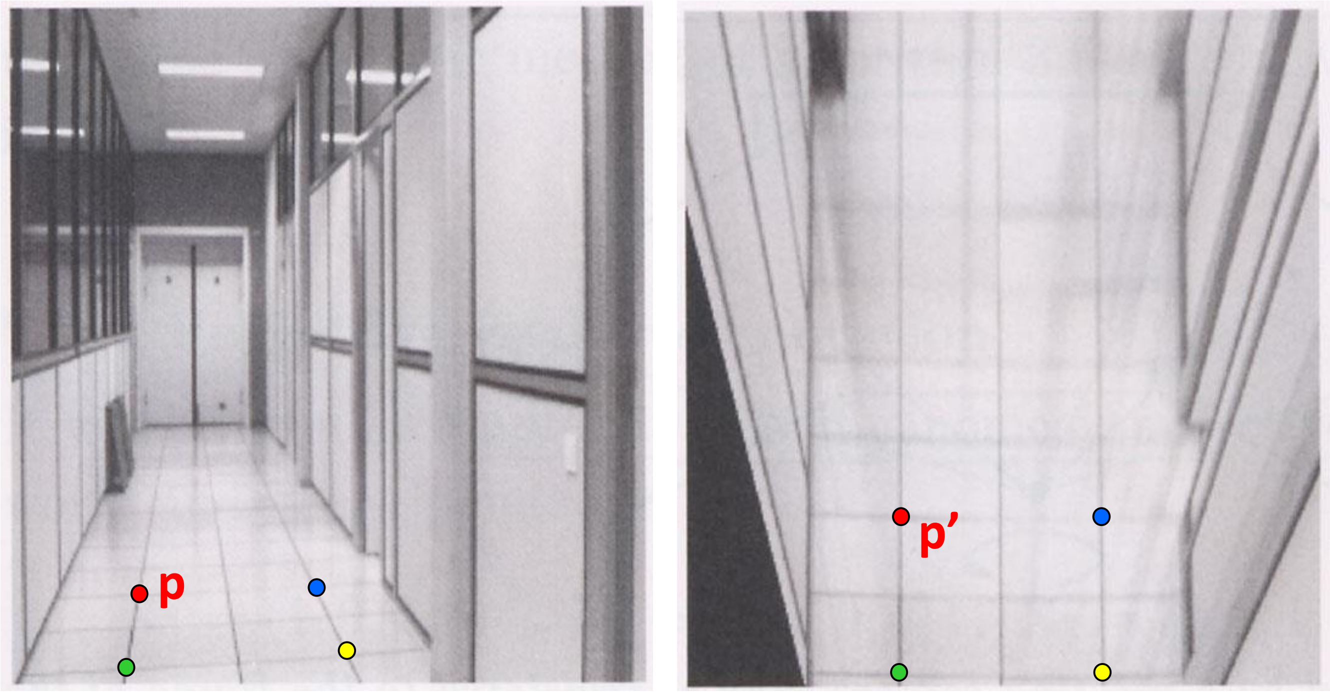


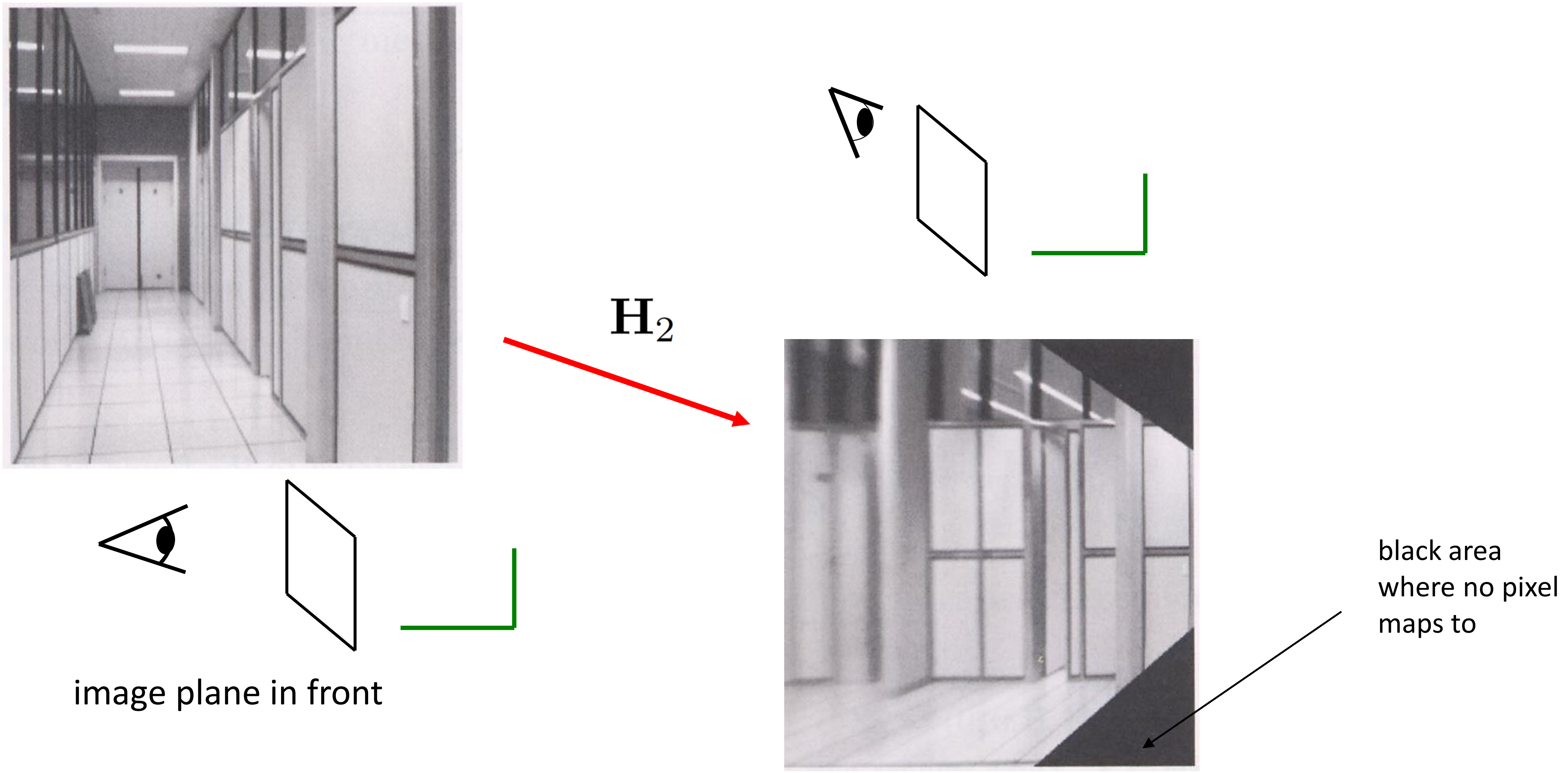
Image rectification



To unwarp (rectify) an image

- Find the homography H given a set of p and p’ pairs
- How many correspondences are needed?
- Tricky to write H analytically, but we can solve for it!
- Find such H that “best” transforms points p into p’
- Use least-squares!

Side view


$\mathbf{H}_2$
image plane in front
black area
where no pixel
maps to

What is the shape of the b/w floor pattern?

Flagellation,
Piero della Francesca

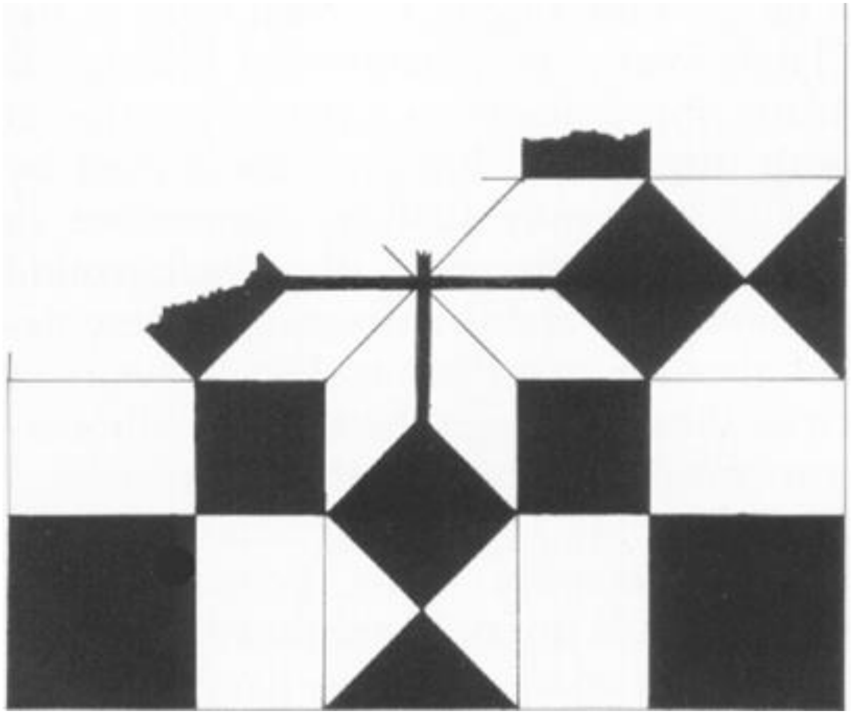
The floor (enlarged)

automatically rectified floor

Slide from Criminisi

Analyzing patterns and shapes

Automatic rectification



From Martin Kemp *The Science of Art* *(manual reconstruction)*

2 patterns have been discovered !

Slide from Criminisi

Analyzing patterns and shapes

St. Lucy Altarpiece, **D. Veneziano**

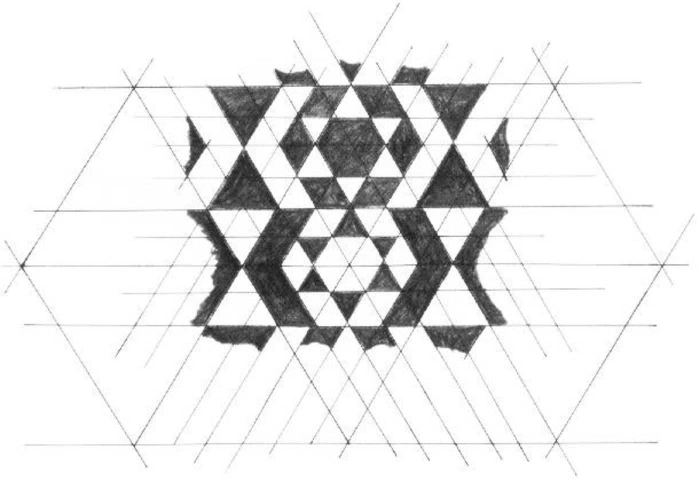
What is the (complicated) shape of the floor pattern?

Automatically rectified floor

Slide from Criminisi

Analyzing patterns and shapes

Automatic rectification



From Martin Kemp, *The Science of Art* *(manual reconstruction)*

Slide from Criminisi

Analyzing patterns and shapes

The Ambassadors by Hans Holbein the Younger, 1533

É una omografia

Omografie

Homographies …

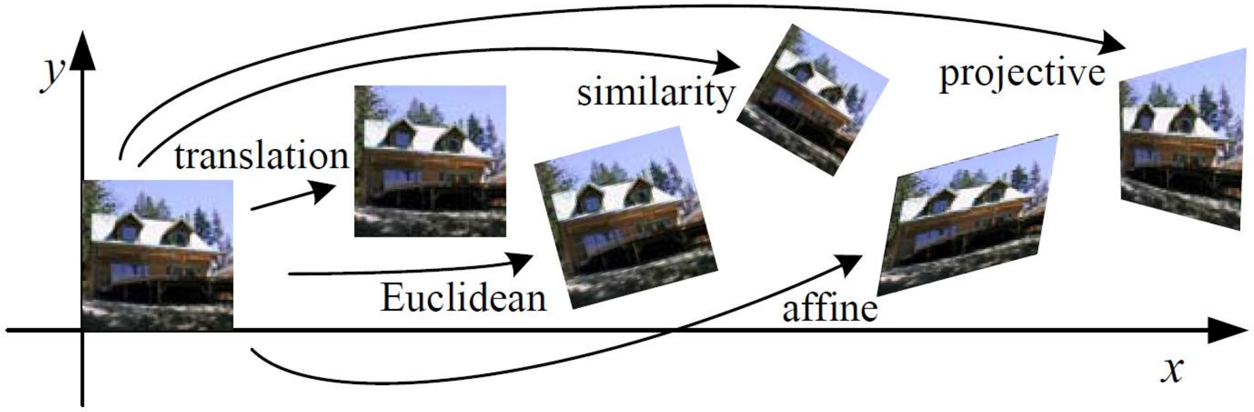
- Affine transformations, and
- Projective warps

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Properties of projective transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Ratios are not preserved
- Closed under composition

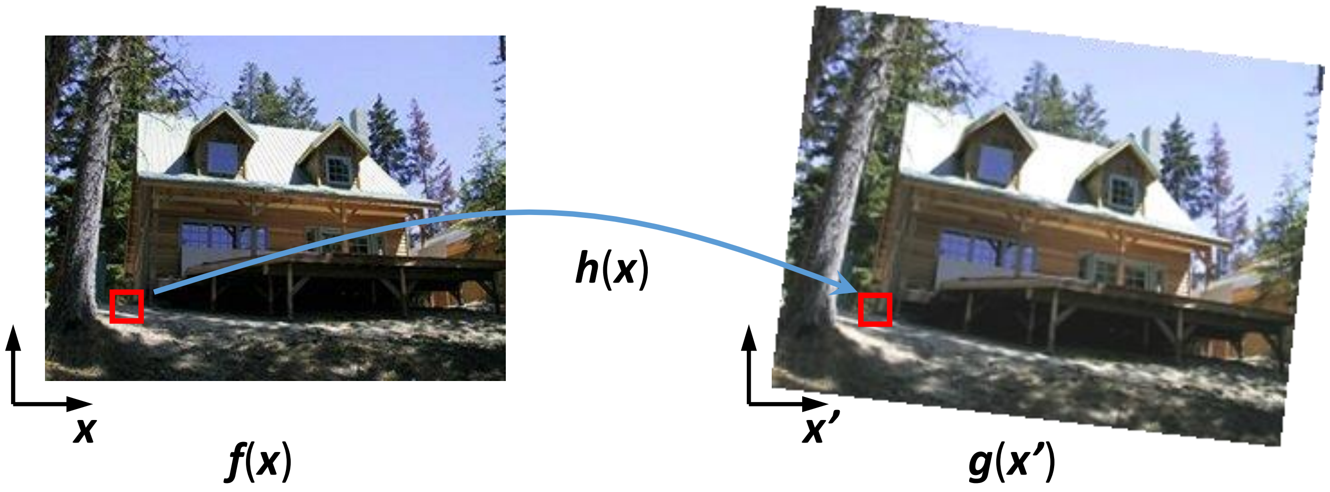
Ricapitolando



Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\left[\begin{array}{c\|c} \boldsymbol{I} & \boldsymbol{t} \end{array}\right]_{2\times 3}$	2	orientation $+\cdots$	
rigid (Euclidean)	$\left[\begin{array}{c\|c} \boldsymbol{R} & \boldsymbol{t} \end{array}\right]_{2\times 3}$	3	lengths $+\cdots$	
similarity	$\left[\begin{array}{c\|c} s\boldsymbol{R} & \boldsymbol{t} \end{array}\right]_{2\times 3}$	4	angles $+\cdots$	
affine	$\left[\begin{array}{c} \boldsymbol{A} \end{array}\right]_{2\times 3}$	6	parallelism $+\cdots$	
projective	$\left[\begin{array}{c} \tilde{\boldsymbol{H}} \end{array}\right]_{3\times 3}$	8	straight lines	

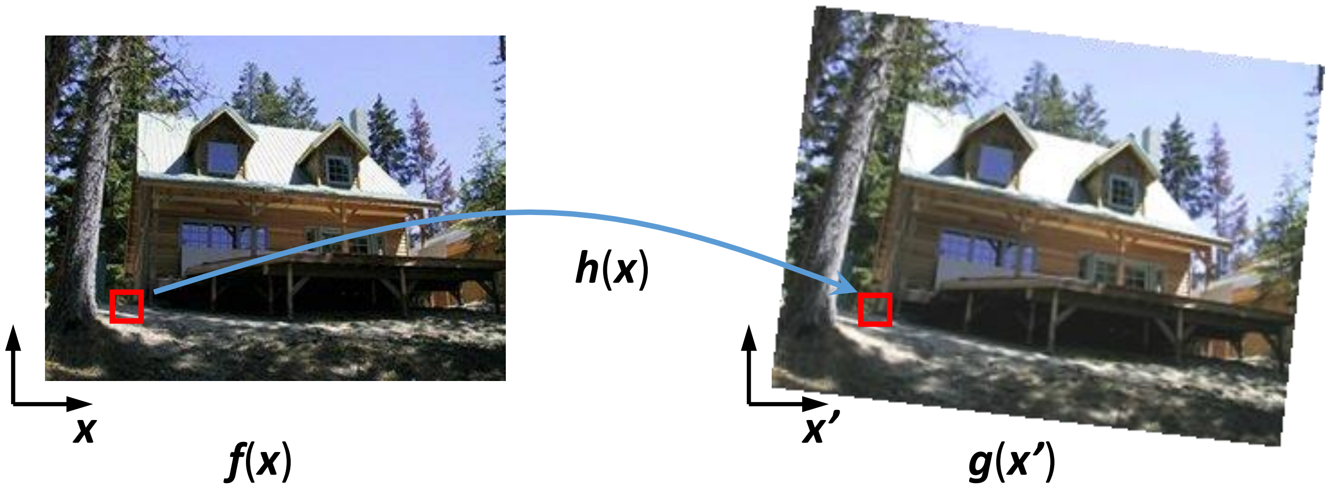
Implementing image warping

Given a coordinate transform $\boldsymbol{x'} = \boldsymbol{h}(\boldsymbol{x})$ and a source image $\boldsymbol{f}(\boldsymbol{x})$, how do we compute a transformed image $\boldsymbol{g}(\boldsymbol{x'}) = \boldsymbol{f}(\boldsymbol{h}(\boldsymbol{x}))$?



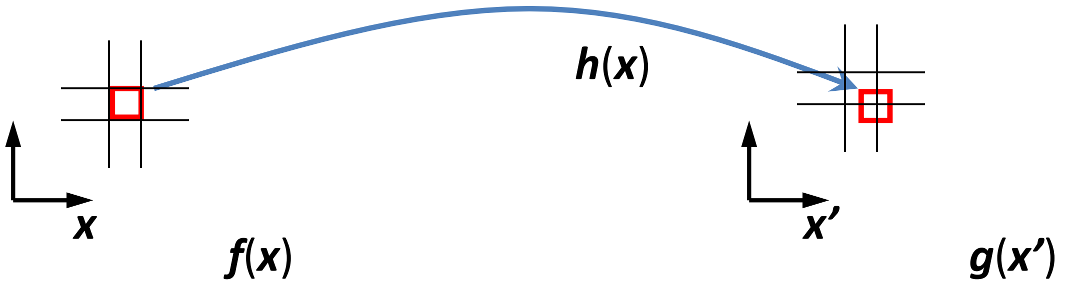
Forward Warping

- Send each pixel ***f***(***x***) to its corresponding location ***x'*** = ***h***(***x***) in ***g***(***x'***)



Forward Warping

- Send each pixel $\boldsymbol{f}(\boldsymbol{x})$ to its corresponding location $\boldsymbol{x'} = \boldsymbol{h}(\boldsymbol{x})$ in $\boldsymbol{g}(\boldsymbol{x'})$
  - What if pixel lands "between" two pixels?
  - Answer: add "contribution" to several pixels, normalize later (*splatting*)

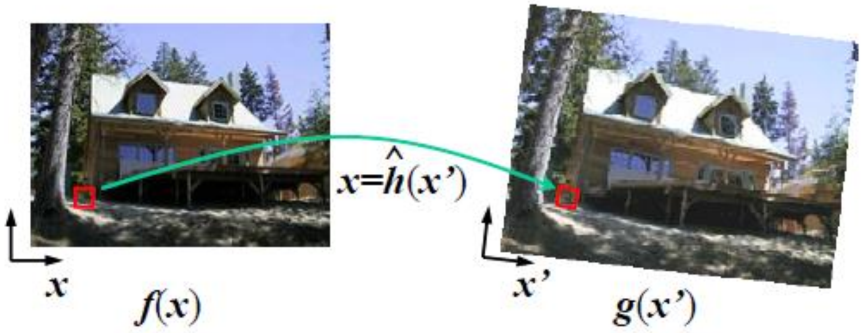


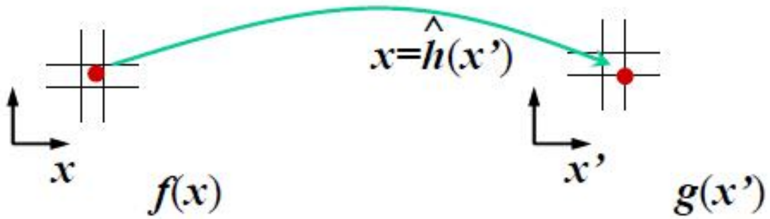
Inverse Warping

procedure *inverseWarp*$(f, \mathrm{h}, \textbf{out } g)$:

For every pixel $\mathbf{x}'$ in $g(\mathbf{x}')$

1. Compute the source location $\mathbf{x} = \hat{\mathbf{h}}(\mathbf{x}')$
2. Resample $f(\mathbf{x})$ at location $\mathbf{x}$ and copy to $g(\mathbf{x}')$

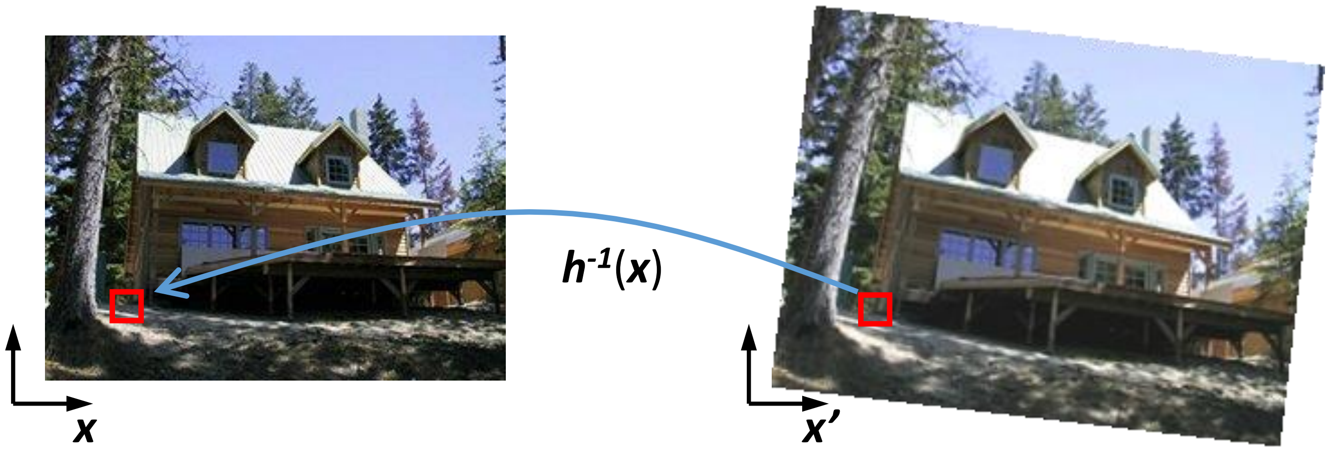




Inverse Warping

Where does the function $\hat{\mathbf{h}}(\mathbf{x}')$ come from? Quite often, it can simply be computed as the inverse of $\mathbf{h}(\mathbf{x})$.

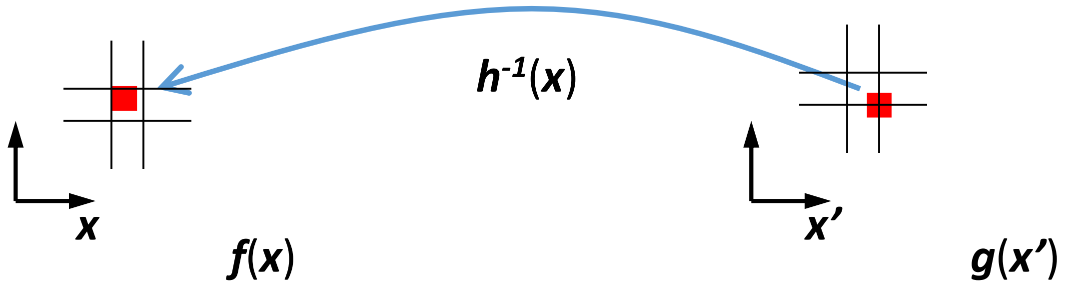
- Requires taking the inverse of the transform
- What if pixel comes from “between” two pixels?



Inverse Warping

Where does the function $\hat{\mathbf{h}}(\mathbf{x}')$ come from? Quite often, it can simply be computed as the inverse of $\mathbf{h}(\mathbf{x})$.

- What if pixel comes from "between" two pixels?
- Answer: *resample* color value from *interpolated* (*prefiltered*) source image



Interpolation

Possible interpolation filters:

- nearest neighbor
- bilinear
- bicubic
- sinc

Needed to prevent "jaggies" and "texture crawl"

(with prefiltering)

Forward vs. inverse warping

Q: Which is better?

A: usually inverse—eliminates holes

- however, it requires an invertible warp function—not always possible...

Esempio omografia+warping

```python
import cv2 as cv
from google.colab.patches import cv2_imshow
from urllib.request import urlopen
import numpy as np

req_left = urlopen('https://dbloisi.github.io/corsi/images/montagna-1.jpg')
arr_left = np.array(bytearray(req_left.read()), dtype=np.uint8)
img_left = cv.imdecode(arr_left, -1)
cv2_imshow(img_left)
```

Esempio omografia+warping

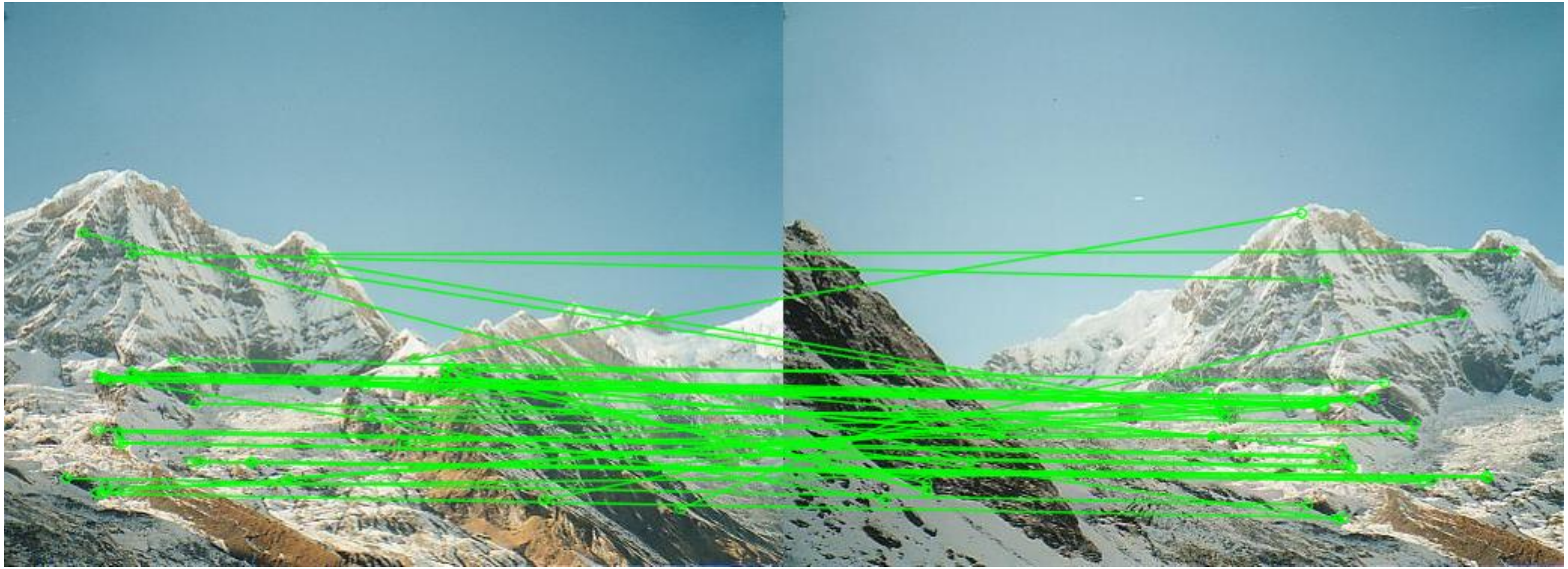
```
req_right = urlopen('https://dbloisi.github.io/corsi/images/montagna-2.jpg')
arr_right = np.array(bytearray(req_right.read()), dtype=np.uint8)
img_right = cv.imdecode(arr_right, -1)
cv2_imshow(img_right)
```

Esempio omografia+warping

```
# orb descriptor
orb = cv.ORB_create()
# find key points
kp1, des1 = orb.detectAndCompute(img_right, None)
kp2, des2 = orb.detectAndCompute(img_left, None)
# brute force matching
match = cv.BFMatcher()
matches = match.knnMatch(des1,des2,k=2)
# distance ratio
# "Distinctive Image Features from Scale-Invariant Keypoints"
# by David G. Lowe
good = []
for m,n in matches:
    if m.distance < 0.85*n.distance:
        good.append(m)

# drawing good matches
draw_params = dict(matchColor=(0,255,0),
                       singlePointColor=None,
                       flags=2)
matches_img = cv.drawMatches(img_right,kp1,img_left,kp2,good,None,**draw_params)
cv2_imshow(matches_img)
```

Esempio omografia+warping



Esempio omografia+warping

```
# homography computation
MIN_MATCH_COUNT = 5
if len(good) > MIN_MATCH_COUNT:
    src_pts = np.float32([ kp1[m.queryIdx].pt for m in good ]).reshape(-1,1,2)
    dst_pts = np.float32([ kp2[m.trainIdx].pt for m in good ]).reshape(-1,1,2)
    M, mask = cv.findHomography(src_pts, dst_pts, cv.RANSAC, 5.0)
    h,w,c = img_right.shape
    pts = np.float32([ [0,0],[0,h-1],[w-1,h-1],[w-1,0] ]).reshape(-1,1,2)
    dst = cv.perspectiveTransform(pts, M)
else:
    print("Not enought matches are found - %d/%d", (len(good)/MIN_MATCH_COUNT))
```

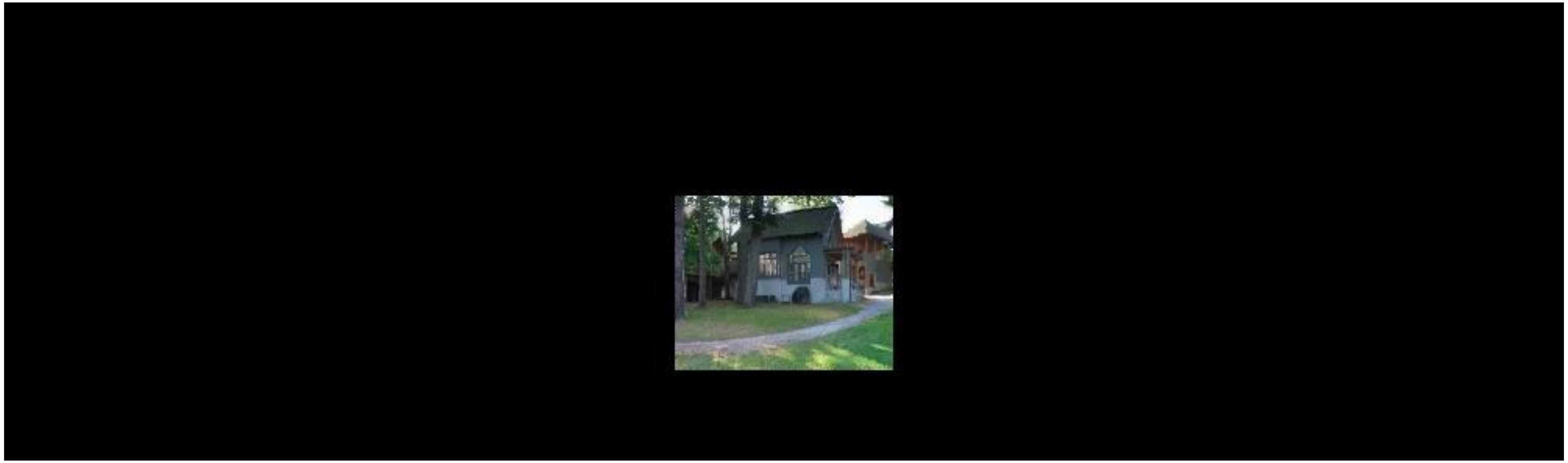
Esempio omografia+warping

```
# creating panorama image
panorama_img = cv.warpPerspective(img_right,M,(img_left.shape[1] + img_right.shape[1], img_left.shape[0]))
panorama_img[0:img_left.shape[0],0:img_left.shape[1]] = img_left
cv2_imshow(panorama_img)
```

Panoramas

Are you getting the whole picture?

Compact Camera FOV = 50 x 35°



Panoramas

Are you getting the whole picture?

Compact Camera FOV = 50 x 35°

Human FOV = 200 x 135°

Panoramas

Are you getting the whole picture?

Compact Camera FOV = 50 x 35°

Human FOV = 200 x 135°

Panoramic Mosaic = 360 x 180°

Why "Recognising Panoramas"?

- 1D Rotations (θ)
  - Ordering $\Rightarrow$ matching images

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- 2D Rotations (θ, ϕ)
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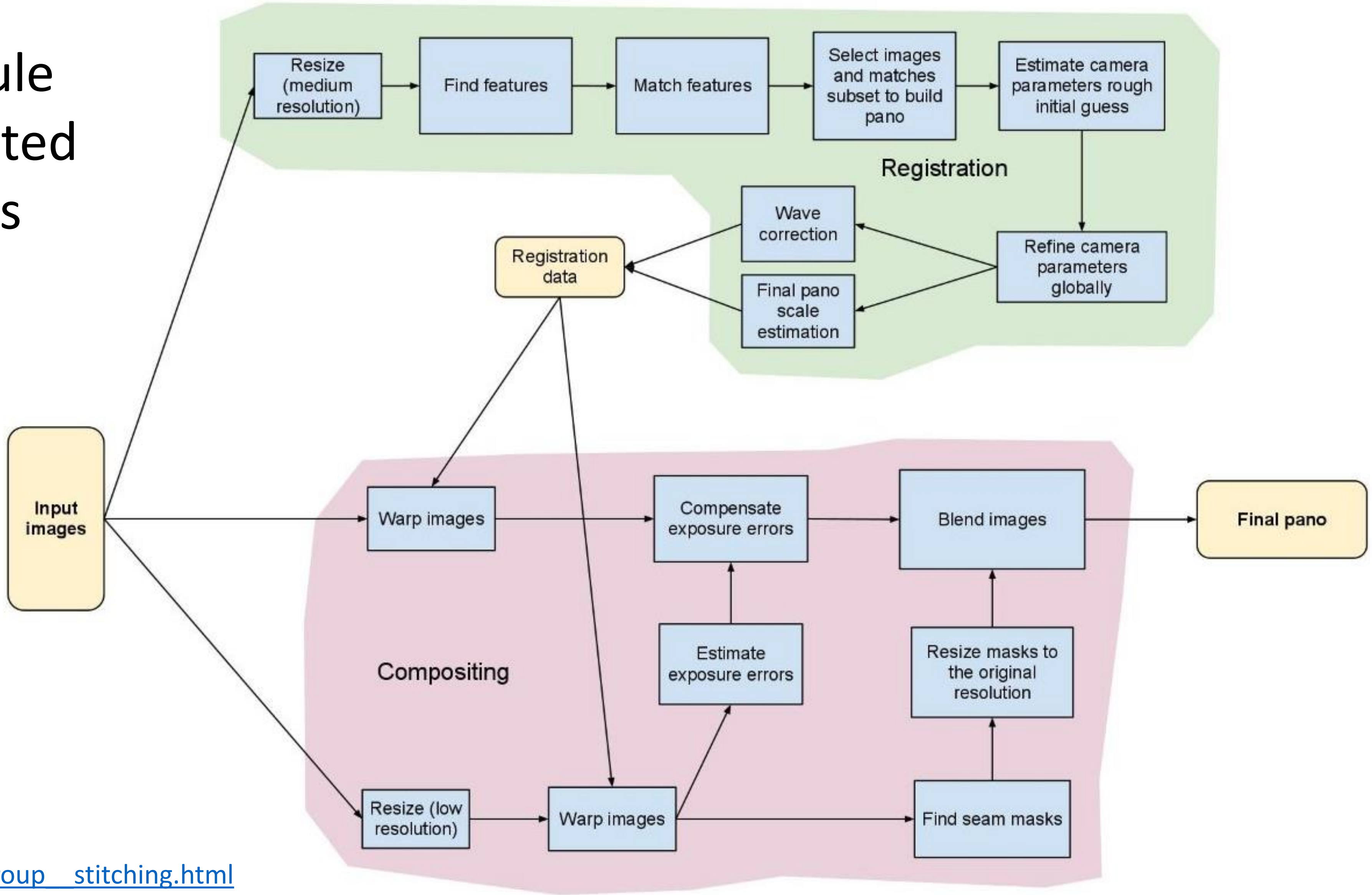
Stitching in OpenCV

The stitching module pipeline implemented in the **Stitcher** class



https://docs.opencv.org/4.1.2/d1/d46/group__stitching.html

Stitching in OpenCV

Stitching in OpenCV

stitching.ipynb

File Edit View Insert Runtime Tools Help

Files

Upload Refresh Mount Drive

..
sample_data
Univ1.jpg
Univ2.jpg
Univ3.jpg
Univ4.jpg
Univ5.jpg

+ Code + Text

```python
import numpy as np
import cv2 as cv
import sys

# read input images
imgs = []
for i in range(1,6):
  img_name = "Univ" + str(i) + ".jpg"
  img = cv.imread(img_name)
  if img is None:
    print("can't read image " + img_name)
    sys.exit(-1)
  else:
    print(img_name + " loaded")
    imgs.append(img)
```

https://docs.opencv.org/4.1.2/d5/d48/samples_2python_2stitching_8py-example.html

Stitching in OpenCV

```
stitcher = cv.Stitcher.create(cv.Stitcher_PANORAMA)

status, pano = stitcher.stitch(imgs)

if status != cv.Stitcher_OK:
  print("Can't stitch images, error code = %d" % status)
  sys.exit(-1)

cv.imwrite("panorama.jpg", pano)

print("stitching completed successfully.")
```

```
stitching completed successfully.
```

https://docs.opencv.org/4.1.2/d5/d48/samples_2python_2stitching_8py-example.html

Stitching in OpenCV

UNIVERSITÀ DEGLI STUDI DELLA BASILICATA

Corso di Visione e Percezione

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Omografie